REINFORCED CROSS SECTION PROPERTIES

REINFORCING PLATE AT TENSION FLANGE

Fig. 1. Beam with a reinforcing plate at the tension flange.

\[ y_{sc} = \frac{I_y d + I_p \left( d + \frac{t_p}{2} \right)}{I_{yb}} \]  

\[ C_w = \frac{I_y h_i^2}{I_r \left( I_y/2 + I_p \right) + 1} \]  

\[ h_i = \frac{(d - t_f/2) b_f t_f + (d + t_p/2) b_p t_p}{b_f t_f + b_p t_p} - \frac{t_f}{2} \]  

\[ r_i = \frac{b_f}{\sqrt{12 \left[ 1 + \frac{t_w (y_c - t_f)}{3 b_f t_f} \right]}} \]  

where

- \( C_w \) = warping constant, in.\(^6\)  
- \( I_p \) = strong-axis moment of inertia of the plate, in.\(^4\)  
- \( I_y \) = moment of inertia of the W shape about the y-axis, in.\(^4\)
\( I_{sb} \) = moment of inertia of the built-up shape about the y-axis, in.\(^4\)
\[
= I_y + I_p
\]

\( b_f \) = flange width of W shape, in.
\( b_p \) = width of plate, in.
\( d \) = depth of W shape, in.
\( h_t \) = distance between centroid of compression flange and centroid of combined plate and tension flange, in.
\( r_l \) = effective radius of gyration for lateral-torsional buckling, in.
\( t_f \) = thickness of flange for W shape, in.
\( t_p \) = thickness of plate, in.
\( t_w \) = thickness of web for W shape, in.
\( y_0 \) = distance from the shear center to the centroid of the built-up shape, in.
\( y_c \) = distance from the face of the compression flange to the centroid of the built-up shape, in.
\( y_{sc} \) = distance from the face of the compression flange to the shear center of the built-up shape, in.
\[
= y_c + y_0
\]
\( y_t \) = distance from the face of the tension flange to the centroid of the built-up shape, in.
REINFORCING PLATE AT EACH FLANGE

Fig. 2. Beam with reinforcing plates at both flanges.

\[ y_{sc} = \frac{I_y \left( t_{pc} + d/2 \right) + I_{pc} \left( \frac{t_{pc}}{2} \right) + I_{pt} \left( d + t_{pc} + \frac{t_{pt}}{2} \right)}{I_{yb}} \]  

(5)

\[ C_w = \frac{I_y}{2} \left[ \left( y_{sc} - t_{pc} - \frac{t_f}{2} \right)^2 + \left( d - y_{sc} + t_{pc} - \frac{t_f}{2} \right)^2 \right] + I_{pc} \left( y_{sc} - \frac{t_{pc}}{2} \right)^2 + I_{pt} \left( d + t_{pc} - y_{sc} + \frac{t_{pt}}{2} \right)^2 \]  

(6)

\[ r_i = \sqrt{\frac{t_f b_f^3 + t_{pc} b_{pc}^3}{12 \left[ b_f t_f + b_{pc} t_{pc} + t_w (y_c - t_f) \right]}} \]  

(7)

where

- \( I_{pc} \) = strong-axis moment of inertia of compression-flange plate, in.\(^4\)
- \( I_{pt} \) = strong-axis moment of inertia of tension-flange plate, in.\(^4\)
- \( I_{yb} \) = moment of inertia of the built-up shape about the y-axis, in.\(^4\)
- \( = I_y + I_{pt} + I_{pb} \)
- \( b_{pc} \) = width of compression-flange plate, in.
- \( b_{pt} \) = width of tension-flange plate, in.
- \( t_{pc} \) = thickness of compression-flange plate, in.
- \( t_{pt} \) = thickness of tension-flange plate, in.
- \( y_c \) = distance from the face of the compression flange to the centroid of the built-up shape, in.
\( y_{sc} \) = distance from the outer face of the compression flange plate to the shear center of the built-up shape, in.
\( = t_{pc} + y_c + y_0 \)

\( y_t \) = distance from the face of the tension flange to the centroid of the built-up shape, in.
TEE REINFORCEMENT AT TENSION FLANGE

Fig. 3. Beam with tee reinforcement at the tension flange.

\[
y_{sc} = \frac{I_y d + I_{yt} \left( d + d_t - \frac{t_{bf}}{2} \right)}{I_{yb}}
\]

(8)

\[
C_w = I_y \left[ \left( y_{sc} - \frac{t_f}{2} \right)^2 + \left( d - y_{sc} - \frac{t_f}{2} \right)^2 \right] + I_{yt} \left( d + d_t - y_{sc} - \frac{t_{bf}}{2} \right)^2
\]

(9)

\[
r_t = \sqrt{\frac{b_f}{12 \left[ 1 + \frac{t_w (y_c - t_f)}{3 b_f t_f} \right]}}
\]

(10)

where

\( I_{yb} \) = moment of inertia of the built-up shape about the y-axis, in.\(^4\)
\( = I_y + I_{yt} \)
\( I_{yt} \) = moment of inertia of the Tee shape about the y-axis, in.\(^4\)
\( b_{bf} \) = width of flange for Tee shape, in.
\( d_t \) = depth of Tee shape, in.
\( t_{bt} \) = thickness of flange for Tee shape, in.
\( t_{wt} \) = thickness of web for Tee shape, in.
\( y_c = \) distance from the face of the compression flange to the centroid of the built-up shape, in.
\( y_{sc} = \) distance from the face of the compression flange to the shear center of the built-up shape, in.
\[ = y_c + y_0 \]
\( y_t = \) distance from the face of the W-shape tension flange to the centroid of the built-up shape, in.
\( y_{tt} = \) distance from the face of the Tee-shape tension flange to the centroid of the built-up shape, in.
Fig. 4. Beam with wide flange reinforcement at the tension flange.

\[ y_{sc} = \frac{I_y d + I_{yr} \left( d + \frac{d_r}{2} \right)}{I_{yb}} \]  

\[ C_w = \frac{I_y}{2} \left[ \left( y_{sc} - \frac{t_f}{2} \right)^2 \right] + \left( d - y_{sc} - \frac{t_f}{2} \right)^2 + \frac{I_{yr}}{2} \left[ \left( d - y_{sc} + \frac{t_f}{2} \right)^2 \right] + \left( d + d_r - y_{sc} - \frac{t_f}{2} \right)^2 \]  

\[ r_i = \frac{b_f}{\sqrt{12 \left[ 1 + \frac{t_w (y_c - t_f)}{3b_f t_f} \right]}} \]

where

- \( I_{yb} \) = moment of inertia of the built-up shape about the y-axis, in.\(^4\)
- \( I_y = I_{yb} \)
- \( I_{yr} \) = moment of inertia of the reinforcing W shape about the y-axis, in.\(^4\)
- \( b_{fr} \) = width of flange for the reinforcing W shape, in.
- \( d_r \) = depth of reinforcing W shape, in.
- \( t_{fr} \) = thickness of flange for reinforcing W shape, in.
- \( t_{wr} \) = thickness of web for reinforcing W shape, in.
- \( y_c \) = distance from the face of the compression flange to the centroid of the built-up shape, in.
\[ y_{sc} = \text{distance from the face of the compression flange to the shear center of the built-up shape, in.} \]
\[ = y_c + y_0 \]
\[ y_t = \text{distance from the face of the reinforced W-shape tension flange to the centroid of the built-up shape, in.} \]
\[ y_{tr} = \text{distance from the face of the reinforcing W-shape tension flange to the centroid of the built-up shape, in.} \]