REINFORCED CROSS SECTION PROPERTIES

REINFORCING PLATE AT TENSION FLANGE

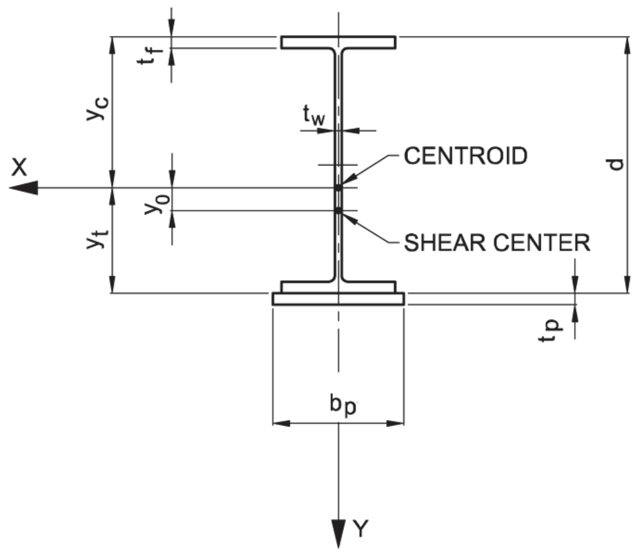

Fig. 1. Beam with a reinforcing plate at the tension flange.

$$y_{sc} = \frac{\dfrac{I_y d}{2} + I_p\left(d + \dfrac{t_p}{2}\right)}{I_{yb}} \tag{1}$$

$$C_w = \frac{I_y h_t^2}{\dfrac{I_y}{I_y/2 + I_p} + 1} \tag{2}$$

$$h_t = \frac{\left(d - t_f/2\right) b_f t_f + \left(d + t_p/2\right) b_p t_p}{b_f t_f + b_p t_p} - \frac{t_f}{2} \tag{3}$$

$$r_t = \frac{b_f}{\sqrt{12\left[1 + \dfrac{t_w\left(y_c - t_f\right)}{3 b_f t_f}\right]}} \tag{4}$$

where

C_w = warping constant, in.^{6}

I_p = strong-axis moment of inertia of the plate, in.^{4}

I_y = moment of inertia of the W shape about the y-axis, in.^{4}

I_{yb} = moment of inertia of the built-up sshape about the y-axis, in.^{4}
= $I_y + I_p$
b_f = flange width of W shape, in.
b_p = width of plate, in.
d = depth of W shape, in.
h_t = distance between centroid of compression flange and centroid of combined plate and tension flange, in.
r_t = effective radius of gyration for lateral-torsional buckling, in.
t_f = thickness of flange for W shape, in.
t_p = thickness of plate, in.
t_w = thickness of web for W shape, in.
y_0 = distance from the shear center to the centroid of the built-up shape, in.
y_c = distance from the face of the compression flange to the centroid of the built-up shape, in.
y_{sc} = distance from the face of the compression flange to the shear center of the built-up shape, in.
= $y_c + y_0$
y_t = distance from the face of the tension flange to the centroid of the built-up shape, in.

REINFORCING PLATE AT EACH FLANGE

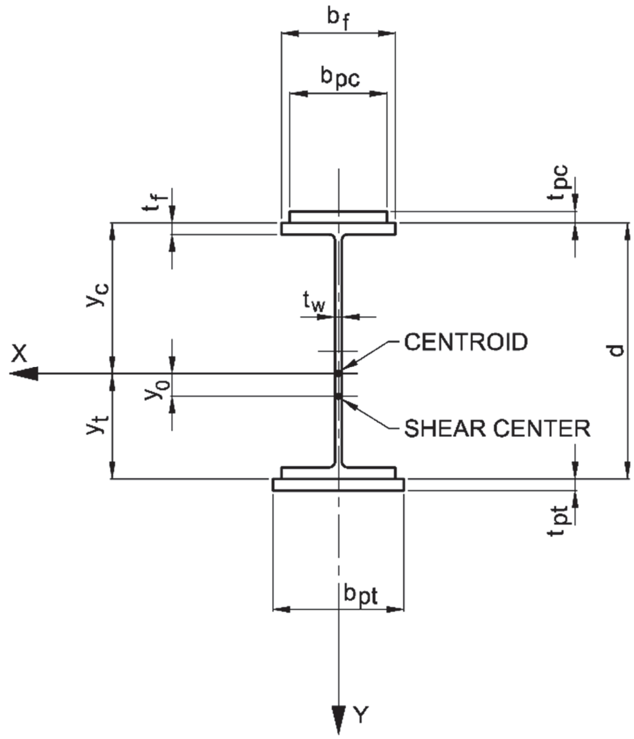

Fig. 2. Beam with reinforcing plates at both flanges.

$$y_{sc} = \frac{I_y \left(t_{pc} + d/2\right) + I_{pc}\left(\frac{t_{pc}}{2}\right) + I_{pt}\left(d + t_{pc} + \frac{t_{pt}}{2}\right)}{I_{yb}} \tag{5}$$

$$C_w = \frac{I_y}{2}\left[\left(y_{sc} - t_{pc} - \frac{t_f}{2}\right)^2 + \left(d - y_{sc} + t_{pc} - \frac{t_f}{2}\right)^2\right] + I_{pc}\left(y_{sc} - \frac{t_{pc}}{2}\right)^2 + I_{pt}\left(d + t_{pc} - y_{sc} + \frac{t_{pt}}{2}\right)^2 \tag{6}$$

$$r_t = \sqrt{\frac{t_f b_f^3 + t_{pc} b_{pc}^3}{12\left[b_f t_f + b_{pc} t_{pc} + t_w \left(y_c - t_f\right)/3\right]}} \tag{7}$$

where

I_{pc} = strong-axis moment of inertia of compression-flange plate, in.^{4}
I_{pt} = strong-axis moment of inertia of tension-flange plate, in.^{4}
I_{yb} = moment of inertia of the built-up shape about the y-axis, in.^{4}
= $I_y + I_{pt} + I_{pb}$
b_{pc} = width of compression-flange plate, in.
b_{pt} = width of tension-flange plate, in.
t_{pc} = thickness of compression-flange plate, in.
t_{pt} = thickness of tension-flange plate, in.
y_c = distance from the face of the compression flange to the centroid of the built-up shape, in.

y_{sc} = distance from the outer face of the compression flange plate to the shear center of the built-up shape, in.
= $t_{pc} + y_c + y_0$

y_t = distance from the face of the tension flange to the centroid of the built-up shape, in.

TEE REINFORCEMENT AT TENSION FLANGE

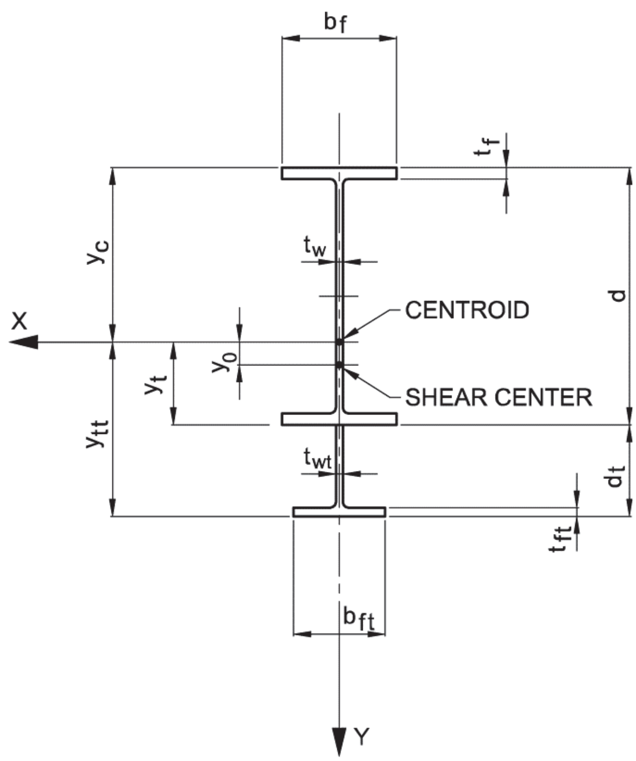

Fig. 3. Beam with tee reinforcement at the tension flange.

$$y_{sc} = \frac{\dfrac{I_y d}{2} + I_{yt}\left(d + d_t - \dfrac{t_{ft}}{2}\right)}{I_{yb}} \tag{8}$$

$$C_w = \frac{I_y}{2}\left[\left(y_{sc} - \frac{t_f}{2}\right)^2 + \left(d - y_{sc} - \frac{t_f}{2}\right)^2\right] + I_{yt}\left(d + d_t - y_{sc} - \frac{t_{ft}}{2}\right)^2 \tag{9}$$

$$r_t = \frac{b_f}{\sqrt{12\left[1 + \dfrac{t_w\left(y_c - t_f\right)}{3b_f t_f}\right]}} \tag{10}$$

where

I_{yb} = moment of inertia of the built-up shape about the y-axis, in.^{4}
= $I_y + I_{yt}$
I_{yt} = moment of inertia of the Tee shape about the y-axis, in.^{4}
b_{ft} = width of flange for Tee shape, in.
d_t = depth of Tee shape, in.
t_{ft} = thickness of flange for Tee shape, in.
t_{wt} = thickness of web for Tee shape, in.

y_c = distance from the face of the compression flange to the centroid of the built-up shape, in.
y_{sc} = distance from the face of the compression flange to the shear center of the built-up shape, in.
= $y_c + y_0$
y_t = distance from the face of the W-shape tension flange to the centroid of the built-up shape, in.
y_{tt} = distance from the face of the Tee-shape tension flange to the centroid of the built-up shape, in.

WIDE FLANGE REINFORCEMENT AT TENSION FLANGE

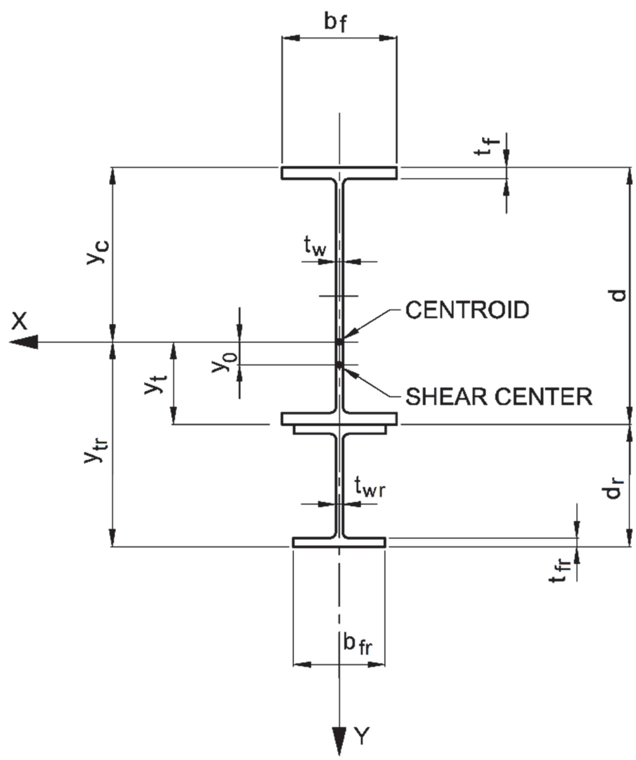

Fig. 4. Beam with wide flange reinforcement at the tension flange.

$$y_{sc} = \frac{\frac{I_y d}{2} + I_{yr}\left(d + \frac{d_r}{2}\right)}{I_{yb}} \tag{11}$$

$$C_w = \frac{I_y}{2}\left[\left(y_{sc} - \frac{t_f}{2}\right)^2 + \left(d - y_{sc} - \frac{t_f}{2}\right)^2\right] + \frac{I_{yr}}{2}\left[\left(d - y_{sc} + \frac{t_{fr}}{2}\right)^2 + \left(d + d_r - y_{sc} - \frac{t_{fr}}{2}\right)^2\right] \tag{12}$$

$$r_t = \frac{b_f}{\sqrt{12\left[1 + \frac{t_w\left(y_c - t_f\right)}{3 b_f t_f}\right]}} \tag{13}$$

where

I_{yb} = moment of inertia of the built-up shape about the y-axis, in.^{4}
= $I_y + I_{yr}$
I_{yr} = moment of inertia of the reinforcing W shape about the y-axis, in.^{4}
b_{fr} = width of flange for the reinforcing W shape, in.
d_r = depth of reinforcing W shape, in.
t_{fr} = thickness of flange for reinforcing W shape, in.
t_{wr} = thickness of web for reinforcing W shape, in.
y_c = distance from the face of the compression flange to the centroid of the built-up shape, in.

y_{sc} = distance from the face of the compression flange to the shear center of the built-up shape, in.
= $y_c + y_0$

y_t = distance from the face of the reinforced W-shape tension flange to the centroid of the built-up shape, in.

y_{tr} = distance from the face of the reinforcing W-shape tension flange to the centroid of the built-up shape, in.